2021

(Held in 2022)

ECONOMICS

Paper : CC–6

(Mathematical Methods in Economics—II)

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. Answer/Choose the correct option of the following : 1×6=6

(a) What do you mean by orthogonal vectors?

(b) State the definition of scalar matrix with example.

(c) If

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}_{2\times 3}$$

then find the transpose of matrix A, i.e., A'.

(d) Find $\frac{dy}{dx}$ of the function $y = 5x^2 - 3$.

(e) Point elasticity of demand (e) is defined as

(i) $\frac{1}{AR - MR}$

(ii) $\frac{AR}{AR - MR}$

(iii) $\frac{1 - AR}{MR}$

(f) In Cobb-Douglas production function

$$Q = AK^{\alpha}L^{\beta}$$

(i) A, α, β parameters are negative

(ii) A, α, β parameters are zero

(iii) A, α, β parameters are positive

2. Answer the following questions : 2×5=10

(a) Define exponential function.

(b) Explain the concept of finite and infinite set.

(c) Prove that if the two rows or two columns are identical, then the value of the determinant will be zero.

(d) What is the difference between homogeneous and homothetic function?

(e) Given the short-run total cost function

$$TC = Q^3 - 3Q^2 + 15Q + 27$$

Obtain the Average Cost (AC) and Marginal Cost (MC) functions.

3. Answer the following questions (any *six*) : 5×6=30

(a) Find all the cofactors of the determinant

$$\begin{vmatrix} 1 & 2 & 3 \\ -4 & 3 & 6 \\ 2 & -7 & 9 \end{vmatrix}_{3\times3}$$

(b) If

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}_{2\times2} \text{ and } B = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}_{2\times2}$$

then prove that $AB = BA = I$.

(c) Calculate the inverse of matrix

$$A = \begin{bmatrix} 2 & -2 & 3 \\ 1 & 0 & -3 \\ 3 & 4 & 0 \end{bmatrix}_{3\times3}$$

(d) Find the equilibrium price ($\bar{P}$) and quantity ($\bar{Q}$) of the following market model :

$$Q_d = 20 - 7P$$
$$Q_s = -4 + 5P$$
$$Q_d = Q_s$$

(e) If the demand law is given by $P = 32 - 3q$, find the elasticity of demand (e_d).

(f) Explain the conditions for maximization and minimization.

(g) The total cost function is given by

$$TC = 100 - 2q + 0 \cdot 5q^2$$

Show that the slope of the AC curve is negative, when the output is less than 10.

(h) State Euler's theorem. Show that the Cobb-Douglas production function

$$Q = f(L, K) = AL^{\beta}K^{1-\beta}$$

satisfies the Euler's theorem, if $\alpha + \beta = 1$.

(i) In a production unit

$$X = 100 + 10K - K^2$$

where K is the input and X is the output. Find the marginal productivity, when $K = 2$.

4. Answer the following questions (any *two*) : 10×2=20

(a) (i) Determine the equilibrium national income ($\bar{Y}$), consumption ($\bar{C}$) and tax ($\bar{T}$) from the following national income model : 7

$$Y = C + I_0 + G_0$$
$$C = \beta(Y - T),\ 1 > \beta > 0$$
$$T = \delta Y,\ 1 > \delta > 0$$

(ii) Find the differential of the function

$$y = \frac{2x^2}{10 + x}$$ 3

(b) (i) Establish the relationship between MR, AR and elasticity of demand (e_d). 5

(ii) If the demand law is given by $P = 16 - 2q$, find elasticity of demand, when $q = 2$. 5

(c) (i) Given

$$Y = f(x_1, x_2) = (x_1^2 + 5x_1x_2^2 + x_2^3)^5$$

Find $\frac{dy}{dx_1}$ and $\frac{dy}{dx_2}$. 5

(ii) A firm's total cost function (TC) is given by $TC = 200 + 5Q + Q^2$ and the total revenue function is $TR = 105Q - 2Q^2$, where Q is the quantity of output. Find out the profit function and the profit maximizing output in terms of Q. 5

5. Answer any *one* of the following questions : 14

(a) (i) Given the total cost function

$$TC = 2Q^2 + 5Q + 18$$

where Q is the output level. Find the output at which average cost is minimum. 4

(ii) Find the optimum value of the function

$$y = 4x_1^3 + 8x_1x_2 - 4x_1^2 - x_2^2 + 10$$ 10

(b) (i) Solve the following system of equations by matrix inversion : 7

$$2x_1 + x_2 + 3x_3 = 15$$
$$x_1 - 2x_2 + 5x_3 = 13$$
$$4x_1 + 3x_2 - x_3 = 11$$

(Continued)

(ii) A monopolist discriminates prices between two markets and the price equations are given by

$$P_1 = 80 - 3Q_1$$
$$P_2 = 104 - 4Q_2$$

where Q_1 and Q_2 are the amount output sold in the two markets. The total cost function of the monopoly firm is given by

$$TC = 50 - 10Q + 2Q^2$$

when $Q = Q_1 + Q_2$. Determine profit maximizing output and maximum profit. 7

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